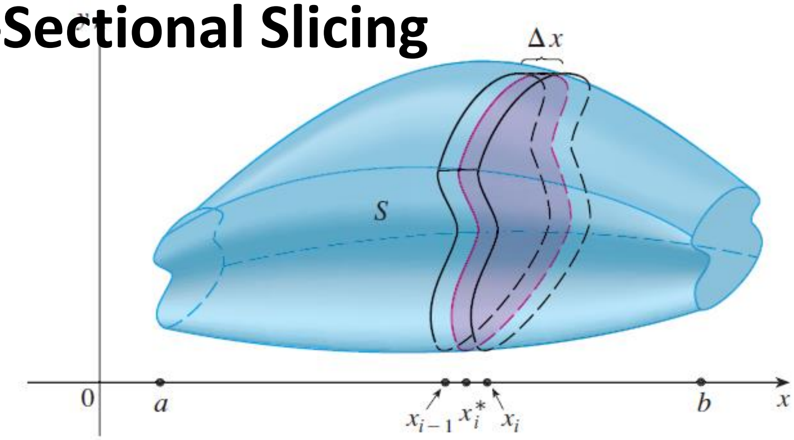
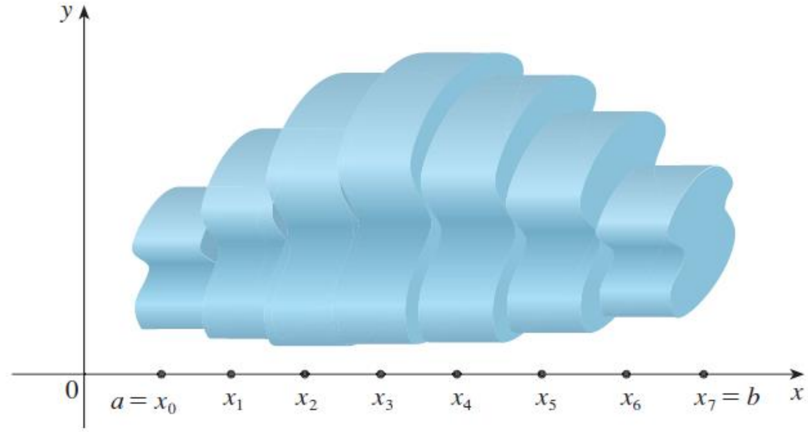
Closing Wed: HW_3A,3B,3C (6.1-6.3)
Exam 1 is Thurs (4.9, 5.1-5.5, 6.1-6.3)

Entry Task:
Find the area of the region bounded by $4x = y^2$ and $y = 2x^3$ in 2 ways:

(i) Using dx
(ii) Using dy

6.2 Finding Volumes Using Cross-Sectional Slicing





If we can find the general formula, $A(x_i)$, for the area of a cross-sectional slice, then we can approximate volume by:

$$\text{Volume of one slice} \approx \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

$$\text{Total Volume} \approx \sum_{i=1}^{n} \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

This approximation gets better and better with more subdivisions, so

$$\text{Exact Volume} = \lim_{n\to\infty} \sum_{i=1}^{n} \mathrm{A}(\mathrm{x}_i)\,\Delta\mathrm{x}$$

We conclude

$$\text{Volume} = \int_a^b A(x)dx =$$

$$\int_a^b \text{"Cross-sectional area formula"} dx$$

Volume using cross-sectional slicing

1. Draw region. Cut **perpendicular** to rotation axis. Label *x* if that cut crosses the *x*-axis (and *y* if *y*-axis). Label **everything** in terms this variable.

2. Formula for cross-sectional area?

   disc: Area = π(radius)^2

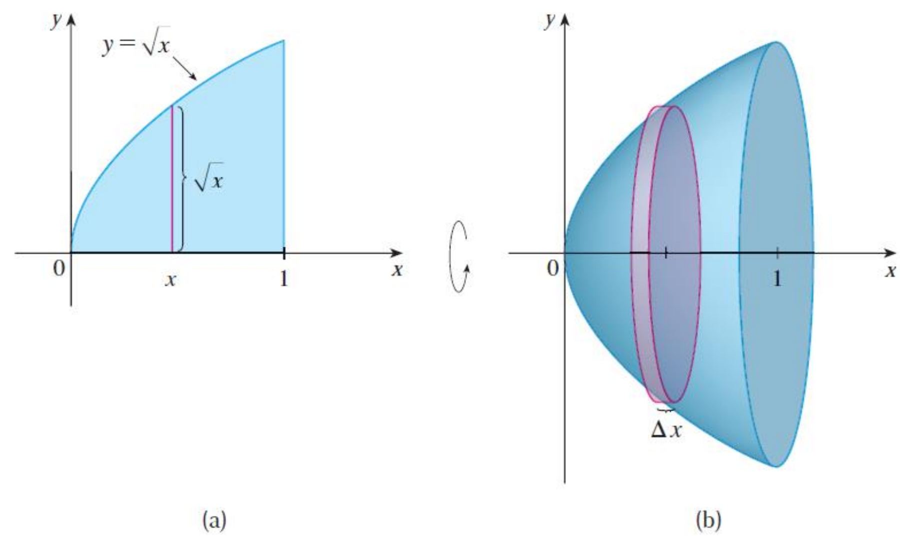
   washer: Area = π(outer)^2 - π(inner)^2

   square: Area = (Height)(Length)

   triangle: Area = ½ (Height)(Length)

3. Integrate the area formula.

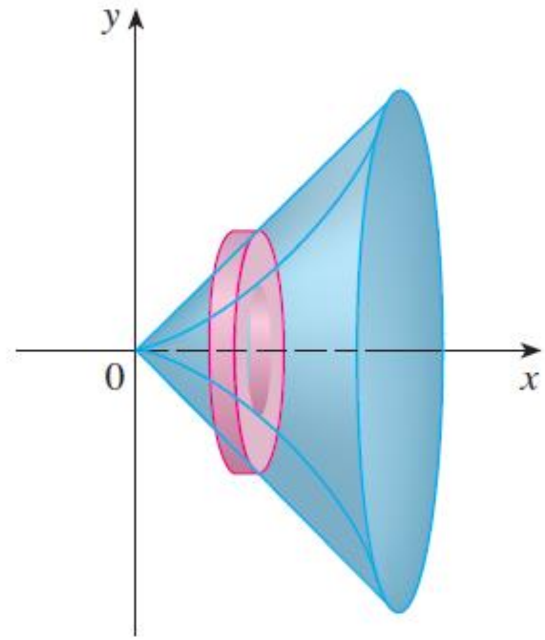
Example: Consider the region, *R*, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the **x-axis**.



Example: Consider the region, R, bounded by $y = \sqrt{x}$, y = 0, and x = 1. Find the volume of the solid obtained by rotating R about the ***y*-axis**.

Example: Consider the region, R, bounded by $y = x$ and $y = x^4$. Find the volume of the solid obtained by rotating R about the **x-axis**.

1. Draw and label!
2. Cross-sectional area?
3. Integrate area.



Example: Consider the region, *R*, bounded by $y = x$ and $y = x^4$. (*R* is the same as the last example).

(a) Now rotate about the horizontal line y = -5. What changes?

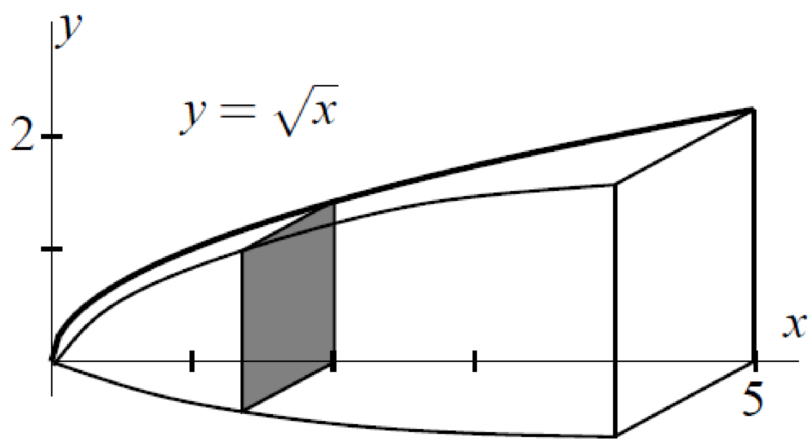
(b) Now rotate about the horizontal line y = 10. What changes?

Example:

(From an old final and homework)

Find the volume of the solid shown.

The cross-sections are squares.



1. Draw and label!
2. Cross-sectional area?
3. Integrate area.

Summary (Cross-sectional slicing):

1. Draw Label
2. Cross-sectional area?
3. Integrate area.

This method has a major limitation:
6.2 method about *x-axis*, must use *dx*.
6.2 method about *y-axis*, must use *dy*.

What if the regions is rotated about the *x*-axis and we need to use *dy*?
(or about *y*-axis and we need dx?)
In these cases, 6.2 “Cross-sectional slicing” wouldn’t work!

We need another method.
That is what we will do in 6.3.